Machine Learning -Practical Implementations-

Time Series Forecasting

ÉCOLE NATIONALE DES PONTS ET CHAUSSÉES

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Outline

Outline

Position of the Problem

- **Goal:** Predict multiple future time steps for a target variable (y_i^t) using:
  - Past observations of the target variable.
  - Additional features that provide context and improve forecasting accuracy.
- **Multiple time series:**
  - We consider several entities $i \in \{1, \ldots, N\}$, each associated with its own time series $(y_i^t)_{t-T_b+1 \leq t \leq t+T_f}$.
  - Examples of entities:
    - **Finance:** Volatility for different stocks in financial markets.
    - **Energy:** Consumption or production across multiple regions.
    - **Traffic:** Flow rates at various locations.

Position of the Problem

- Let us consider an entity i at time t:
- We aim to predict the future values of the univariate time series $(y_i^t)_{t-T_b+1 \leq t \leq t+T_f}$:
  - The past values in a T_b sized window of the target time series: $(y_i^{t-T_b+1}, \ldots, y_i^t)$
  - The future values up to the horizon T_f: $(y_i^{t+1}, \ldots, y_i^{t+T_f})$
- There are 3 possible inputs:

Name	**Notation**
Static attributes	$s_i \in \mathbb{R}^{d_s}$
Time varying unknown	$(z_i^{t-T_b+1}, \ldots, z_i^t) \in \mathbb{R}^{T_b \times d_z}$
Time varying known	$(x_i^{t+1}, \ldots, x_i^{t+T_f}) \in \mathbb{R}^{T_f \times d_x}$

Table: Types of Inputs

Position of the Problem

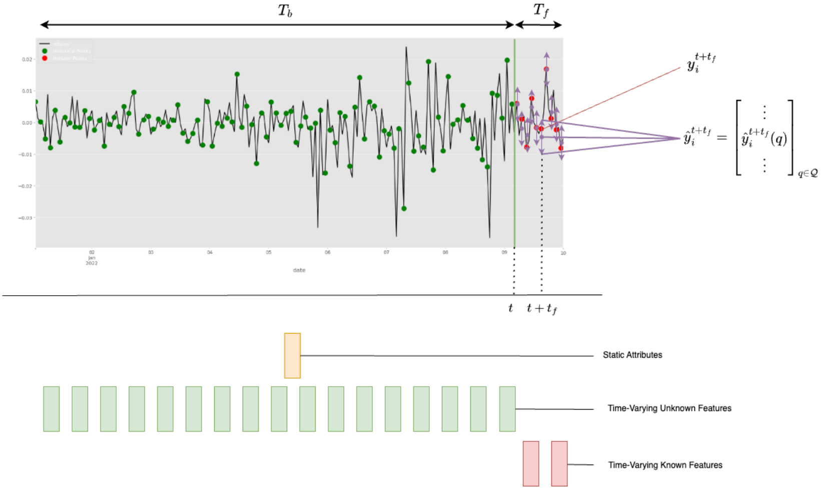
- **Features for Prediction:**
  - **Static Attributes:**
    - Fixed characteristics of each financial asset.
    - Example: Industry sector or market capitalization of a stock.
  - **Time-Varying Known Features:**
    - Features whose future values are available or predictable.
    - Example: Economic calendar events, such as interest rate decisions or earnings announcements.
  - **Time-Varying Unknown Features:**
    - Sequential features observed only up to the present time.
    - Example: Recent trends in stock price movements or realized volatility.

Position of the Problem

- Let $\mathcal{Q}$ be the set of quantiles that interest us. For this example, $\mathcal{Q} = \{0.1, 0.5, 0.9\}$.
- The model outputs for each time step $t + t_f$ (for $t_f \in \{1, \ldots, T_f\}$) the prediction associated with each quantile $q \in \mathcal{Q}$, denoted as $\hat{y}_i^{t+t_f}(q)$.
- Thus, for each $t_f \in \{1, \ldots, T_f\}$, the output vector at each time step $t + t_f$ is given by:

$$\hat{y}_i^{t+t_f} = \begin{bmatrix} \vdots \\ \hat{y}_i^{t+t_f}(q) \\ \vdots \end{bmatrix}_{q \in \mathcal{Q}}$$

Example: The following graph summarizes the previous notations:



The Learning Problem

- To train the model, we compare the predictions $\hat{y}_i^{t+t_f} \in \mathbb{R}^{|\mathcal{Q}|}$ to the true values $y_i^{t+t_f}$ for all $t_f \in \{1, \ldots, T_f\}$.
- The loss function is defined as:

$$\mathcal{L}(\mathcal{B}, \theta) = \sum_{i \in \mathcal{B}} \sum_{q \in \mathcal{Q}} \sum_{t_f=1}^{T_f} \frac{QL_q\left(y_i^{t+t_f}, \hat{y}_i^{t+t_f}(q)\right)}{|\mathcal{B}|\, T_f}$$

- Where:
  - $\mathcal{B}$ is the batch of training data.
  - $\forall y, \hat{y} \in \mathbb{R}$, $QL_q(y, \hat{y}) = q(y - \hat{y})_+ + (1 - q)(\hat{y} - y)_+$
  - Equivalently:

$$QL_q(y, \hat{y}) = \max\left((q-1)(y-\hat{y}), q(y-\hat{y})\right)$$

Outline

Position of the Problem

Temporal Processing using RNNs

The Transformer Architecture

The Variable Selection Network

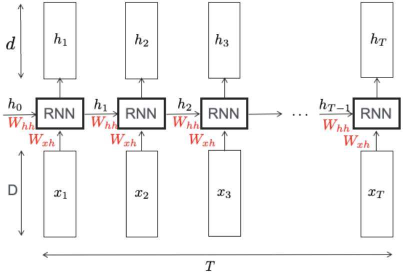
The TFT Architecture

Programming Session: Forecasting daily realized volatility of 31 stock indices

From Hidden Markov Models to RNNs

- **Hidden Markov Models (HMMs)**:
  - Popular in the 1980s for sequence modeling (e.g., speech recognition [10]).
  - Relied on the Markov assumption for hidden states, limiting their ability to model long-range dependencies.
- **Recurrent Neural Networks (RNNs)**:
  - Introduced to overcome HMM limitations.
  - Achieved state-of-the-art performance in tasks such as speech recognition [4].

- Feed-forward neural networks assume data is independent and identically distributed (i.i.d).
- Recurrent Neural Networks (RNNs) [11] process data sequentially, making them suitable for time-series and other sequence-based tasks.



Vanilla RNN Architecture

- Objective: Process an input sequence of D-dimensional vectors $x_1, \ldots, x_T$ to generate d-dimensional hidden states $h_1, \ldots, h_T$.
- Model Parameters:
  - $W_{xh} \in \mathbb{R}^{D \times d}$: Input-to-hidden weights.
  - $W_{hh} \in \mathbb{R}^{d \times d}$: Hidden-to-hidden weights.
- Hidden state at time t:

$$h_t = \tanh\left(W_{hh}^T h_{t-1} + W_{xh}^T x_t\right)$$

Gradient Problems

- **Exploding Gradients:**
  - Occur when gradients become excessively large, destabilizing model training.
- **Vanishing Gradients:**
  - Occur when gradients diminish during backpropagation, preventing the model from learning long-term dependencies.
  - Often observed in deep or sequential networks when dealing with long input sequences.

Addressing Gradient Problems

- **Solutions to Exploding Gradients:**
  - Gradient Clipping: Caps gradients to stabilize training [9].
- **Solutions to Vanishing Gradients:**
  - **Regularization:** Preserves norm consistency during training [9].
  - **Gated Architectures:**
    - Long Short-Term Memory (LSTM) [5]: Introduces gates to manage information flow.
    - Gated Recurrent Unit (GRU) [3]: Simplified alternative to LSTM.

Overview of LSTMs

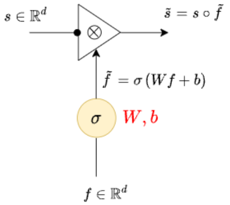
- LSTMs were state-of-the-art for tasks like:
  - Machine Translation [13, 3, 1].
  - Language Modeling [12].
  - Time Series Prediction [6].
  - Robot Reinforcement Learning [2].
- Core Idea: Maintain long-term dependencies through a **cell state** regulated by **gates**.
- Gates are responsible for filtering information flow:
  - Input: New information to add.
  - Forget: Remove irrelevant information.
  - Output: Decide what to expose to the hidden state.

The Concept of Gates in LSTMs

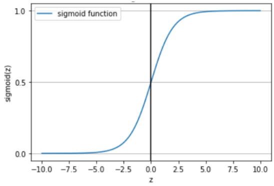
- Gates use a sigmoid function to scale values between 0 and 1:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

- Point-wise multiplication adjusts information based on gate values.



(a) Filtering a signal using a sigmoid function and a neural network



(b) The sigmoid function

- Each time step has:
  - **Cell State** C^t: Preserves long-term memory.
  - **Hidden State** h^t: Represents short-term output.
- Transition from (h^{t-1}, C^{t-1}) to (h^t, C^t) involves:
  1. Filtering with input and forget gates.
  2. Generating a memory candidate $\tilde{C}^t$.
  3. Updating the cell state and computing the hidden state.

The LSTM Architecture

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  - **Cell State** C^t: Preserves long-term memory.
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- Transition from (h^{t-1}, C^{t-1}) to (h^t, C^t) involves:
  1. Filtering with input and forget gates.
  2. Generating a memory candidate $\tilde{C}^t$:

$$\tilde{C}^t = \tanh\left(W_C[h^{t-1}, x^t] + b_C\right)$$

  3. Updating the cell state and computing the hidden state.

LSTM Gate Operations

- **Forget Gate:** Filters irrelevant past memory.

$$f^t = \sigma\left(W_f[h^{t-1}, x^t] + b_f\right)$$

- **Input Gate:** Filters new memory candidate.

$$i^t = \sigma\left(W_i[h^{t-1}, x^t] + b_i\right)$$

- **Output Gate:** Determines visible parts of the cell state.

$$o^t = \sigma\left(W_o[h^{t-1}, x^t] + b_o\right)$$

Memory Update and Hidden State

- **Cell State Update:**

$$C^t = f^t \circ C^{t-1} + i^t \circ \tilde{C}^t$$

- **Hidden State Update:**

$$h^t = o^t \circ \tanh(C^t)$$

- Result: LSTMs Handle long-term dependencies better than vanilla RNNs. LSTMs can:
  - **Write:** Add new information via the input gate.
  - **Erase:** Remove irrelevant information via the forget gate.
  - **Read:** Expose relevant memory via the output gate.

The LSTM Architecture

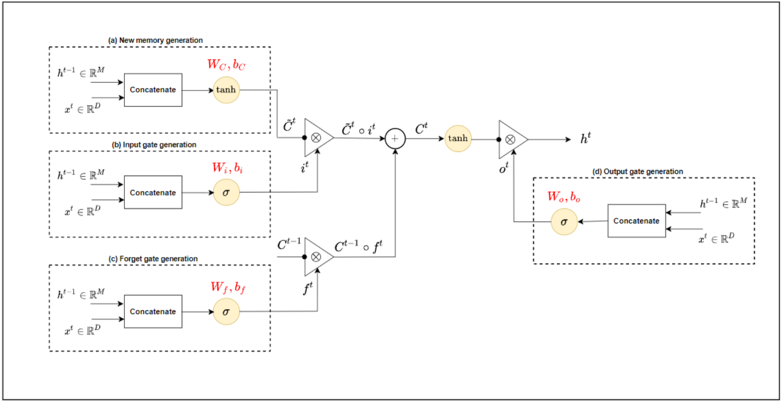
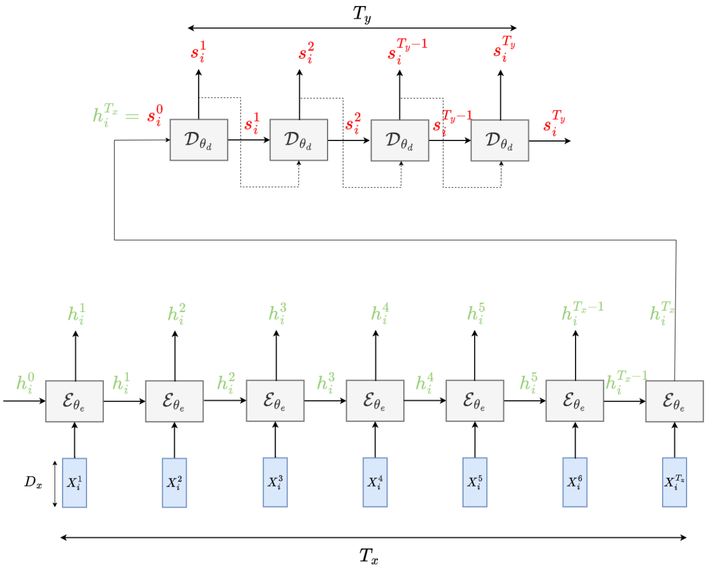


Figure: LSTM architecture.

Sequence to Sequence Framework

- LSTM/GRU models in Many-to-Many settings require input and output sequences of the same length (e.g., POS tagging [15]).
- For applications where $T_x \neq T_y$ (e.g., machine translation), we need the Sequence to Sequence (Seq2Seq) framework.
- Seq2Seq maps an input sequence of length T_x to an output sequence of length T_y using two components:
  1. **Encoder:** Encodes the input sequence into a fixed-length representation.
  2. **Decoder:** Generates the output sequence from the encoded representation.

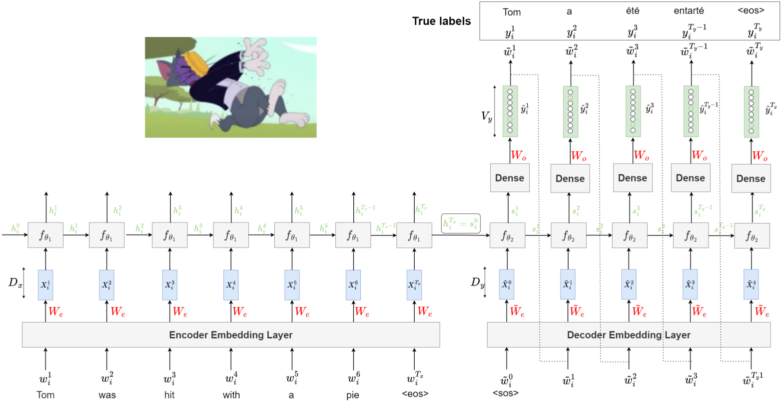
Sequence to Sequence Framework



Encoder and Decoder Components

- **Encoder:**
  - Maps the input sequence $(X_i^1, \ldots, X_i^{T_x}) \in \mathbb{R}^{T_x \times D_x}$ into hidden states $h_i^1, \ldots, h_i^{T_x}$.
  - Final hidden state $h_i^{T_x}$ summarizes the input sequence.
- **Decoder:**
  - Takes the encoder's last hidden state $h_i^{T_x}$ as its initial hidden state s_i^0.
  - Generates the output sequence $s_i^1, \ldots, s_i^{T_y}$.

Example: Seq2seq for machine translation



Limitations of Classical Models

- **Challenges with Seq2Seq Framework:**
  - Encoder compresses all input information into a fixed-length vector, leading to information loss.
  - Performance degrades for long input sequences.
  - No mechanism for aligning input and output sequences.
- **Alignment Intuition:**
  - For each output Y_i^t, the model should selectively focus on relevant parts of the input sequence $X_i^{t'}$.
  - Alignment helps determine how much of each $X_i^{t'}$ contributes to generating Y_i^t.

The Need for Alignment

- The following figure shows the desired alignment matrix, where scores indicate the relevance of each input vector to a specific output.

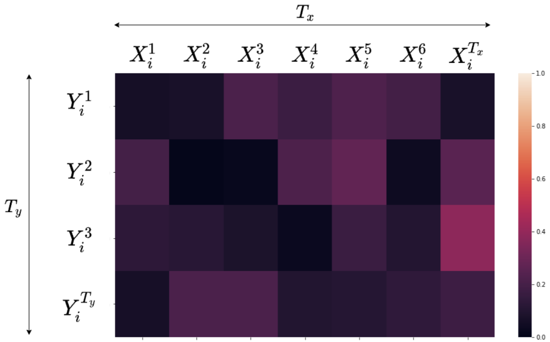


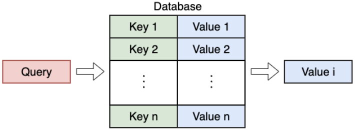
Figure: Matrix of alignment scores.

Addressing the Challenges with Attention

- **Key Challenges of Seq2Seq:**
  - No explicit mechanism to focus on relevant parts of the input.
- **Why Attention?**
  - Allows models to dynamically focus on relevant input parts.
  - Combines perception with a selective memory mechanism for reasoning.
- **Applications:**
  - Machine translation.
  - Time series prediction.
  - Speech-to-text.

Attention: Query-Retrieval Modeling

- Attention mechanisms are inspired by database Query-Retrieval Problems:
  - A query is matched against keys to retrieve values.
  - The following figure shows a classic hard query retrieval system.



- In attention mechanisms:
  - Multiple keys can match a query (**soft query retrieval**).
  - The result is a weighted sum of values, called the **attention vector**.

Soft Query Retrieval: Steps

- Given: a query $q \in \mathbb{R}^{d_q}$, keys $(k_i)_{1 \leq i \leq n} \in \mathbb{R}^{n \times d_k}$, and values $(v_i)_{1 \leq i \leq n} \in \mathbb{R}^{n \times d_v}$.
- Steps:
  1. Compute alignment scores a_i between the query and each key:
  $$a_i = a(q, k_i) \quad \forall i \in \{1, \ldots, n\}.$$
  2. Normalize scores to get attention weights α_i using a distribution function (e.g., softmax):
  $$\alpha_i = \frac{e^{a_i}}{\sum_{j=1}^{n} e^{a_j}}.$$
  3. Compute the attention vector as a weighted sum of values:
  $$A(q, K, V) = \sum_{i=1}^{n} \alpha_i v_i.$$

Alignment Functions

- Alignment functions compute similarity between query q and keys k_i:

Function	**Equation**
Dot Product	$a(q, k_i) = q^T k_i$
Scaled Dot Product	$a(q, k_i) = \frac{q^T k_i}{\sqrt{d_k}}$
Luong's Multiplicative	$a(q, k_i) = q^T W k_i$
Bahdanau's Additive	$a(q, k_i) = v_a^T \tanh(W_1 q + W_2 k_i)$
Feature-based	$a(q, k_i) = W_{imp}^T \mathrm{act}(W_1 \phi_1(k_i) + W_2 \phi_2(q) + b)$
Kernel Method	$a(q, k_i) = \phi(q)^T \phi(k_i)$

Table: Common Alignment Functions.

Soft and Sparse Attention

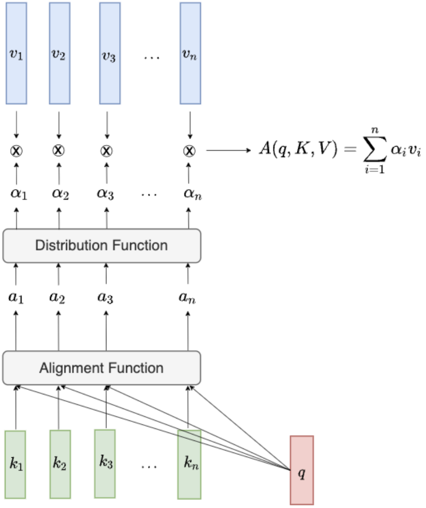
- **Soft Attention:**
  - Uses dense alignments with a softmax function:

$$\alpha_i = \frac{e^{a_i}}{\sum_{j=1}^{n} e^{a_j}}.$$

- **Sparse Attention:**
  - Assigns non-zero probabilities to only a few values.
  - Examples:
    - Sparsemax [7].
    - Sparse Entmax [8].
- The attention vector combines weighted values:

$$A(q, K, V) = \sum_{i=1}^{n} \alpha_i v_i.$$

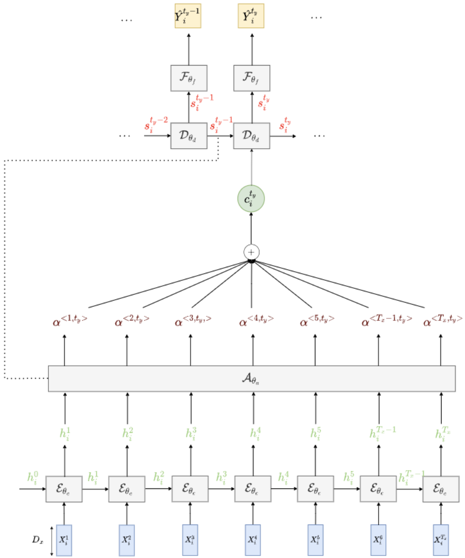
Soft Query Retrieval Summary



- Objective: Learn a mapping function Φ_θ from input sequences to output sequences.

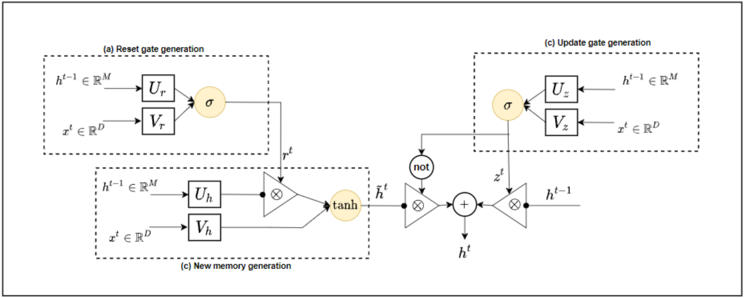
$$(\hat{Y}_i^1, \ldots, \hat{Y}_i^{T_y}) = \Phi_\theta(X_i^1, \ldots, X_i^{T_x}).$$

- Components of Φ_θ:
  1. **Encoder:** Maps input sequence to hidden states $h_i^1, \ldots, h_i^{T_x}$.
  2. **Attention Layer:** Computes context vector $c_i^{t_y}$ for each output step.
  3. **Decoder:** Generates output sequence using attention and decoder states.



- The encoder can be a GRU model or an LSTM model that transforms input sequence $(X_i^1, \ldots, X_i^{T_x})$ into hidden states $(h_i^1, \ldots, h_i^{T_x})$.

- The GRU Model



The Attention Layer

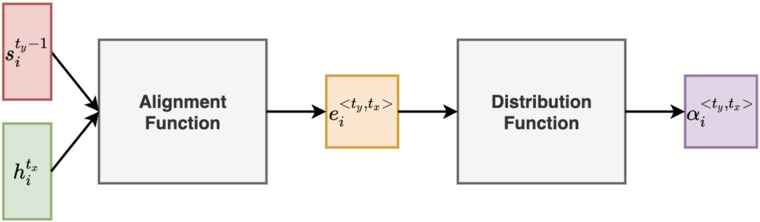
- Assigns weights to encoder hidden states to compute a **context vector** $c_i^{t_y}$:

$$c_i^{t_y} = \sum_{t_x=1}^{T_x} \alpha_i^{<t_y,t_x>} h_i^{t_x}.$$

- Steps:
  1. Compute alignment scores $e_i^{<t_y,t_x>}$ between decoder hidden state $s_i^{t_y-1}$ and encoder hidden states $h_i^{t_x}$.
  2. Normalize scores into attention weights $\alpha_i^{<t_y,t_x>}$ using a distribution function (e.g., softmax).
  3. Calculate context vector $c_i^{t_y}$ as a weighted sum of encoder hidden states.

The Attention Layer

- **Calculating the weights**: $\alpha_i^{<t_y,t_x>}$ for all $t_x \in \{1, \ldots, T_x\}$:



The Decoder and Application-Specific Final Layer

- **Decoder:** Combines:
  - Previous hidden state $s_i^{t_y-1}$,
  - Context vector $c_i^{t_y}$ (from the attention mechanism),
  - To generate the decoder hidden state $s_i^{t_y}$.
- **Final Layer:** Maps $s_i^{t_y}$ to the output prediction $\hat{Y}_i^{t_y}$.
  - The nature of the final layer depends on the application:
    - **Machine Translation:** Dense layer with a softmax activation to predict the next word in a target language.
    - **Text Generation:** Softmax-based layer for generating characters or tokens.
    - **Time Series Forecasting:** Regression output layer for predicting continuous values, such as stock prices or energy consumption.

Outline

Introduction to the Transformer

- The paper **"Attention is All You Need"** [14] introduced a groundbreaking model called the **Transformer**.
- Key contributions:
  - Eliminates the need for recurrent units (e.g., RNNs, LSTMs) in sequence-to-sequence tasks.
  - Fully relies on self-attention mechanisms for capturing dependencies.
- The Transformer model has revolutionized sequence modeling tasks such as machine translation, text summarization, and more.

Introduction to the Transformer

- The following figure illustrates the full Transformer architecture.

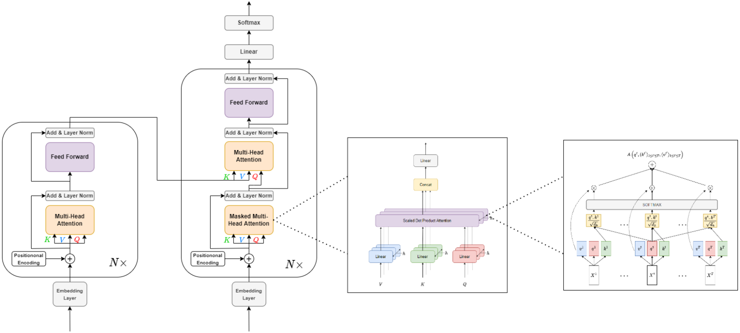
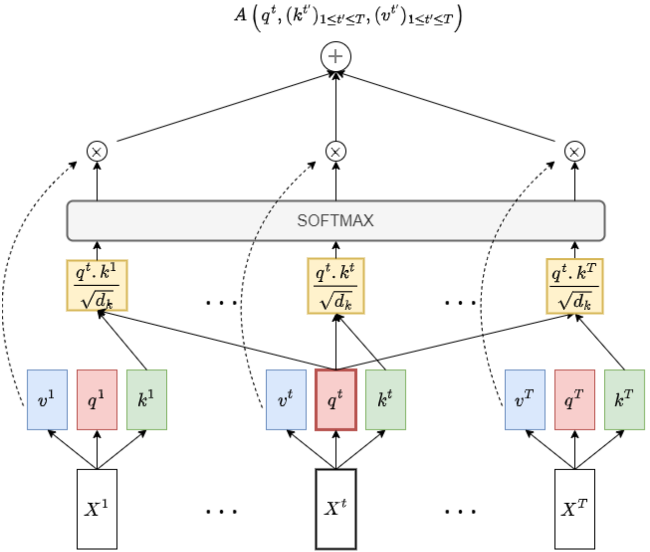


Figure: The Transformer Architecture [14].

Creating a Contextual Embedding with Self-Attention

- Given a sequence of D-dimensional input vectors $(X^t)_{1 \leq t \leq T}$, we project each vector X^t into:
  - Query space: $q^t = W_Q^T X^t, \; W_Q \in \mathbb{R}^{D \times d_q}$,
  - Key space: $k^t = W_K^T X^t, \; W_K \in \mathbb{R}^{D \times d_k}$,
  - Value space: $v^t = W_V^T X^t, \; W_V \in \mathbb{R}^{D \times d_v}$.
- **Objective**: Create a **contextual embedding** for each query q^t, leveraging all keys $(k^{t'})_{1 \leq t' \leq T}$ and values $(v^{t'})_{1 \leq t' \leq T}$.
- **Intuition**: Compute the attention weights $\alpha^{<t,t'>}$ to determine how much each value $v^{t'}$ contributes to the embedding $A\left(q^t, (k^{t'})_{1 \leq t' \leq T}, (v^{t'})_{1 \leq t' \leq T}\right)$.

Creating a Contextual Embedding with Self-Attention



Computing the Contextual Embedding

- Use the **scaled dot product alignment function** [14] to compute similarity scores:

$$e^{<t,t'>} = \frac{q^t \cdot k^{t'}}{\sqrt{d_k}}$$

- Convert similarity scores to attention weights using the softmax distribution:

$$\alpha^{<t,t'>} = \frac{e^{<t,t'>}}{\sum_{s=1}^{T} e^{<t,s>}}$$

- Compute the **contextual embedding**:

$$A\left(q^t, (k^{t'})_{1 \leq t' \leq T}, (v^{t'})_{1 \leq t' \leq T}\right) = \sum_{t'=1}^{T} \alpha^{<t,t'>} v^{t'}$$

Scaled Dot Product Attention Matrix

- To compute contextual embeddings for all input vectors $(X^t)_{1 \leq t \leq T}$, we define:

$$Q = \begin{bmatrix} q^1 \\ \vdots \\ q^T \end{bmatrix} \in \mathbb{R}^{T \times d_q}, \; K = \begin{bmatrix} k^1 \\ \vdots \\ k^T \end{bmatrix} \in \mathbb{R}^{T \times d_k}, \; V = \begin{bmatrix} v^1 \\ \vdots \\ v^T \end{bmatrix} \in \mathbb{R}^{T \times d_v}.$$

- Each q^t, k^t, v^t is computed using projection matrices:

$$q^t = W_Q^T X^t, \quad k^t = W_K^T X^t, \quad v^t = W_V^T X^t.$$

- Q, K, V represent the query, key, and value matrices, respectively.

Scaled Dot Product Attention Matrix

- Definition:

$$A(Q, K, V) := \text{Softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right) V.$$

- Explanation:
  - $\frac{QK^T}{\sqrt{d_k}}$: Computes pairwise similarities between queries and keys.
  - Softmax: Converts similarities into attention weights.
  - Multiplication with V: Aggregates values using attention weights.
- Each row of $A(Q, K, V)$ corresponds to:

$$A(q^t, K, V) = \sum_{t'=1}^{T} \alpha^{<t,t'>} v^{t'}, \quad \forall t \in \{1, \dots, T\}.$$

MultiHead Attention (MHA)

- **Objective:** Extend the attention mechanism to multiple heads to capture diverse notions of similarity.
- Attention mechanism is applied h times:

$$A\left(QW_Q^{h'}, KW_K^{h'}, VW_V^{h'}\right) \quad \text{for } h' \in \{1, \ldots, h\}.$$

- Projection matrices for each head h':

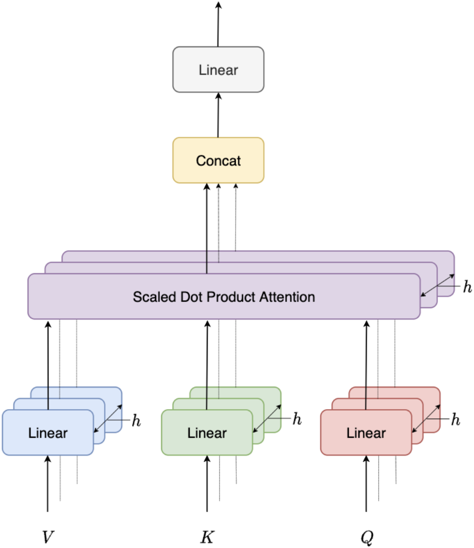
$$W_Q^{h'} \in \mathbb{R}^{d_q \times p_q}, \quad W_K^{h'} \in \mathbb{R}^{d_k \times p_k}, \quad W_V^{h'} \in \mathbb{R}^{d_v \times p_v}.$$

- Outputs are concatenated and projected:

$$P = \text{concat}\left(A_1, \ldots, A_h\right) W_o \in \mathbb{R}^{T_q \times p_o}.$$

Scaled Dot Product Attention Generalization

- The following figure illustrates MHA with h attention heads



Positional Encoding: Intuition and Key Idea

- **Objective:** Incorporate positional information into the permutation-invariant attention mechanism to reflect the order of sequence elements.
- **Intuition:**
  - Positions in a sequence need a unique representation to differentiate elements based on their location.
  - Shifting a positional encoding by k steps results in a consistent transformation that preserves relative distances.
- **Key Idea:**
  - Add positional encoding vectors $p^1, \dots, p^T \in \mathbb{R}^D$ to input embeddings $X^1, \dots, X^T$.
  - Use periodic functions (sine and cosine) to define the positional encodings in a way that captures relative positions effectively.

Positional Encoding: Method and Properties

- **Method:**
  - Positional encoding at step t:

$$p_d^t = \begin{cases} \sin(w_d t), & \text{if } d \text{ is odd}, \\ \cos(w_d t), & \text{if } d \text{ is even}. \end{cases}$$

    Where $w_d = \frac{1}{10000^{\frac{2d}{D}}}$ ensures unique frequencies for different dimensions.
  - Positional encodings are added to input embeddings:

$$\tilde{X}^t = X^t + p^t.$$

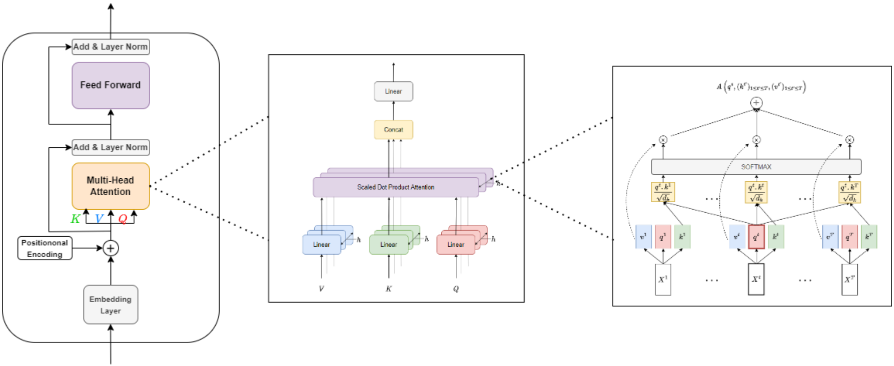
- **Properties:**
  - **Shift Consistency:** Shifting p^t by k steps aligns with p^{t+k}.
  - **Relative Distance Encoding:** The sine and cosine functions ensure relative positional information is preserved across sequences.

The Transformer Architecture

- **Sequence-to-Sequence Model:** Built entirely on attention mechanisms, eliminating recurrent units.
- **Core Components:**
  - **Multi-Head Attention Layer:** Captures different notions of similarity.
  - **Feed-Forward Layer:** Applies pointwise transformations.
  - **Normalization Layer:** Ensures stability and accelerates convergence.
- **Architecture Overview:** Combines stacked encoder and decoder layers to process and generate sequences efficiently.

The Encoder Layer

- **Objective:** Generate attention-based contextual embeddings that focus on relevant parts of the input sequence.
- **Structure:**
  - Stack of $N = 6$ identical layers.
  - Each layer consists of:
    - **Multi-Head Self-Attention:** Re-averages value vectors for contextual embeddings.
    - **Feed-Forward Layer:** Fully connected, applied pointwise.
    - **Residual Connections and Normalization:** Added after each sub-layer.
- **Output Dimension:** $d_{\text{model}} = 512$ for all layers.


Add & Layer Norm
Feed Forward
Add & Layer Norm
Multi-Head Attention
K
V
Q
Positiononal Encoding
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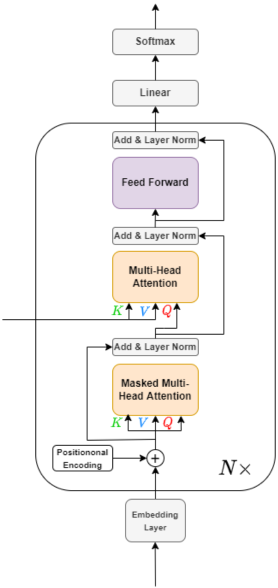
Linear
Concat
Scaled Dot Product Attention
h
Linear
Linear
Linear
V
K
Q
SOFTMAX

The Decoder Layer

- **Objective:** Retrieve and use information from encoder outputs to generate target sequences.
- **Structure:**
  - Stack of $N = 6$ identical layers.
  - Each layer includes:
    - **Masked Multi-Head Self-Attention:** Prevents information leakage (look-ahead masking).
    - **Multi-Head Attention:** Queries the encoder outputs.
    - **Feed-Forward Layer:** Applies pointwise transformations.
    - **Residual Connections and Normalization:** Enhance gradient flow and stability.

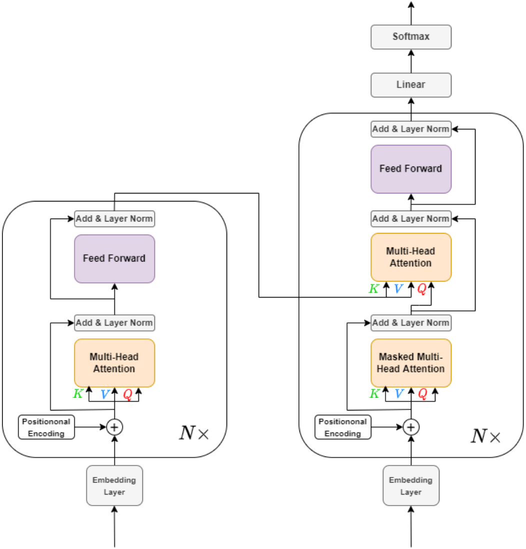
The Decoder Layer in the Transformer



The Complete Transformer Architecture

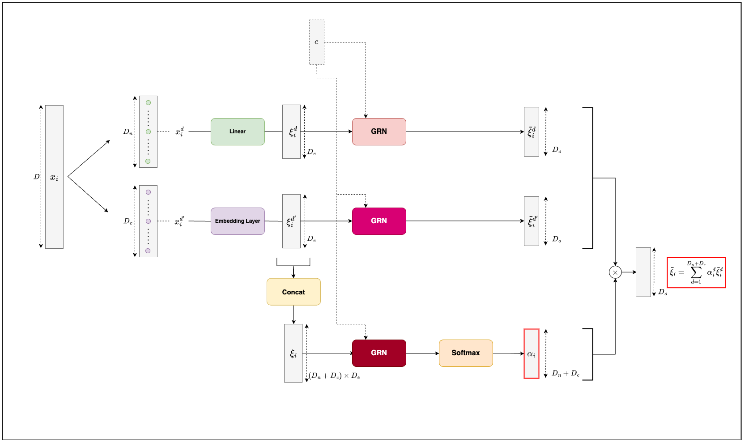
- **Input Processing:**
  - Input sequence $X = (X^1, \ldots, X^{T_x})$ embedded and combined with positional encodings.
  - Encoder outputs context-aware representations $H = (h^1, \ldots, h^{T_x})$.
- **Decoding Process:**
  - Decoder uses:
    - **Self-Attention:** Processes previously generated tokens with masked attention.
    - **Encoder-Decoder Attention:** Focuses on encoder outputs H to generate context for predictions.
  - Outputs generated step-by-step using linear and softmax layers.

The Transformer Architecture



Outline

- **Explore the Variable Selection Network (VSN):** Click here for the detailed implementation



Outline

Position of the Problem

Temporal Processing using RNNs

The Transformer Architecture

The Variable Selection Network

The TFT Architecture

Programming Session: Forecasting daily realized volatility of 31 stock indices

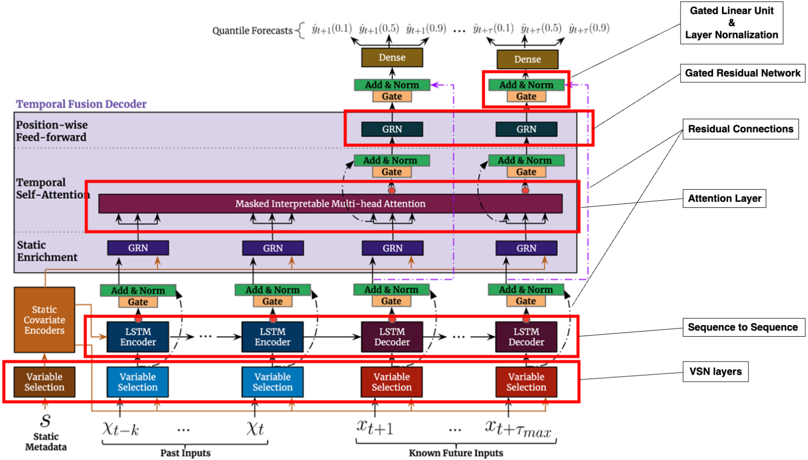
Temporal Fusion Transformer Architecture

- **Overview:** A specialized deep learning model for time series forecasting with the following components:
  - **Variable Selection Networks (VSN):**
    - Dynamically select the most relevant features from static, time-varying known features, and time-varying unknown features.
    - Employ Gated Residual Networks for feature transformation and importance estimation.
  - **Sequence-to-Sequence Framework:**
    - Encoder-decoder architecture for multi-step forecasting.
    - Encoder processes historical data, while the decoder generates future predictions.

Temporal Fusion Transformer Architecture

- **Masked Multi-Head Attention:**
  - Enables context-aware forecasting by focusing on relevant time steps in the past.
  - Prevents information leakage by masking future time steps during decoding.
- **Gated Residual Networks (GRN):**
  - Adds non-linear transformations and flexible gating mechanisms.
  - Regularizes and improves robustness across diverse datasets.

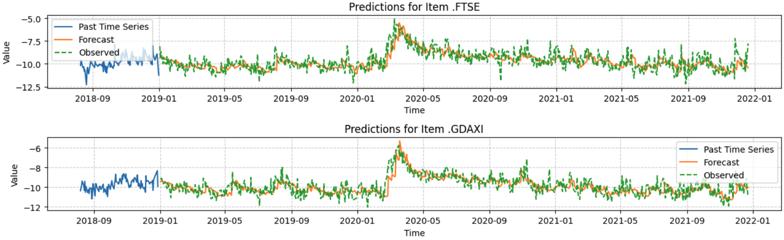
TFT Architecture: High-Level View



Outline

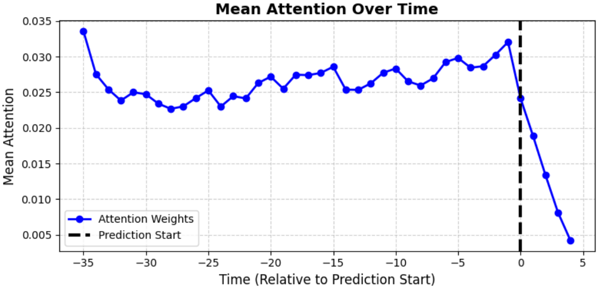
- **Objective:** Implement and experiment with the Temporal Fusion Transformer (TFT) for forecasting realized volatility.
- **Dataset:** Realized volatility data from 31 financial indices.
- **Goals:**
  - Build the TFT model architecture.
  - Train the model on time series data with static, time-varying known, and time-varying unknown features.
  - Evaluate predictions and interpret model outputs.

- **Task:** Forecast realized volatility for 31 indices.
- **Outcome**: Example of predicted vs. actual realized volatility for two indices.



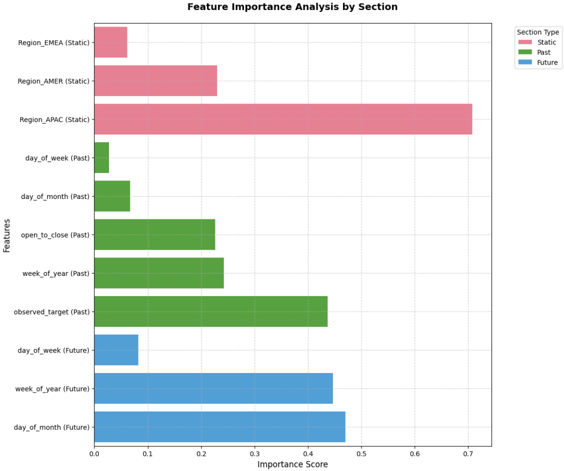
Results: Attention Weights

- **Objective:** Understand how the model uses historical data for forecasting by highlighting the most influential historical time steps.



Results: Feature Importance

- **Objective:** Quantify the impact of input features on predictions.



Thank you for your attention

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