Data Structures and Algorithms
with applications in Machine Learning
- MCQ 2 -

NAME: ______________________ **GROUP:** ______________________

Each Question: 1 Mark **Duration:** 20 Minutes

Completely fill the circles as shown: ○○●○

Answer sheet

Q1. ○ a.
○ b.
○ c.
○ d.

Q2. ○ a.
○ b.
○ c.
○ d.

Q3. ○ a.
○ b.
○ c.
○ d.

Q4. ○ a.
○ b.
○ c.
○ d.

Q5. ○ a.
○ b.
○ c.
○ d.

Q6. ○ a.
○ b.
○ c.
○ d.

Q7. ○ a.
○ b.
○ c.
○ d.

Q8. ○ a.
○ b.
○ c.
○ d.

Q9. ○ a.
○ b.
○ c.
○ d.

Q10. ○ a.
○ b.
○ c.
○ d.

The Quiz

Q. 1 According to the definition of an instability in a matching:

○ a. A pair (m, w) is unstable if the man m prefers another woman w' over w, but w' does not prefer m over her current match.

○ b. A matching is unstable if there are participants left unmatched.

○ c. A matching is unstable if every pair (m, w) prefers their current partners over all others.

● d. A pair (m, w) is unstable if both m and w prefer each other over their current matches.

Q. 2 A matching $\phi : \mathcal{M} \to \mathcal{W}$ is considered stable if:

○ a. Every participant is matched to exactly one partner, regardless of preferences.

● b. There is no instability, meaning no pair (m, w) exists such that m and w prefer each other over their current matches.

○ c. No man m prefers any woman w' over his current match, regardless of w''s preference.

○ d. The matching ensures that every participant is paired with their least preferred valid partner.

Q. 3 Regarding the *laissez-faire* approach to resolving instabilities:

○ a. It guarantees that the process will always terminate with a stable matching.

○ b. It resolves all instabilities simultaneously, ensuring optimal efficiency.

● c. It resolves instabilities iteratively but does not guarantee convergence to a stable matching.

○ d. It avoids resolving instabilities and instead focuses on random pairings.

Q. 4 Which of the following statements about the Gale-Shapley algorithm is true?

○ a. The Gale-Shapley algorithm always results in the most optimal matching for both men and women simultaneously.

○ b. The Gale-Shapley algorithm terminates only if there are no instabilities but does not guarantee a stable matching.

● c. The Gale-Shapley algorithm guarantees a stable matching for any set of preferences.

○ d. The Gale-Shapley algorithm ensures a matching, but stability depends on the initial preferences.

Q. 5 The following pseudo-code implements the Gale-Shapley algorithm:

```
**Algorithm 1** Gale-Shapley Algorithm
**Require:** Lists of preferences for men $\mathcal{M}$ and women $\mathcal{W}$
**Ensure:** Stable matching $\phi$
All men and women start as free
while $\exists$ free man $m$ who has not proposed to every woman on his list do
    Pick such a man $m$
    Let $w$ be the next woman on $m$'s preference list
    if ________ then                                   ▷ Fill in the blank
        Engage $m$ and $w$
    else
        $w$ rejects $m$
    end if
end while
```

What should replace the blank in the condition?

● a. w is free or prefers m to her current match

○ b. w is free and prefers m to her current match

○ c. w prefers her current match to m

○ d. w is free but has already rejected m

Q. 6 The following function implements the Gale-Shapley stable matching algorithm.

```
def gale_shapley(men_prefs, women_prefs):
    """
    Implementation of Gale-Shapley stable matching algorithm.
    """
    n = len(men_prefs)
    free_men = list(range(n))
    proposals = np.zeros(n, dtype=int)
    women_partners = [-1] * n

    while free_men:
        m = free_men[0]
        w = men_prefs[m][proposals[m]]
        proposals[m] += 1

        if women_partners[w] == -1:
            women_partners[w] = m
            free_men.pop(0)
        else:
            current_m = women_partners[w]
            if __________:  # Fill in the blank
                women_partners[w] = m
                free_men.pop(0)
                free_men.append(current_m)

    matches = {(m, w) for w, m in enumerate(women_partners)}
    return matches
```

What should replace the blank?

○ a. `list(women_prefs[m]).index(w) < list(women_prefs[m]).index(current_m)`

● b. `list(women_prefs[w]).index(m) < list(women_prefs[w]).index(current_m)`

○ c. `m > current_m`

○ d. `current_m == -1`

Q. 7 Consider the following preference lists for 4 men and 4 women:

```
men_prefs = [
    [0, 1, 2, 3],  # Man 0's preferences
    [2, 0, 1, 3],  # Man 1's preferences
    [1, 3, 0, 2],  # Man 2's preferences
    [0, 2, 3, 1]   # Man 3's preferences
]

women_prefs = [
    [2, 1, 3, 0],  # Woman 0's preferences
    [0, 3, 1, 2],  # Woman 1's preferences
    [1, 0, 3, 2],  # Woman 2's preferences
    [0, 1, 2, 3]   # Woman 3's preferences
]
```

What is the output of the Gale-Shapley algorithm (as a set of stable matches)?

○ a. {`(0, 0), (1, 2), (2, 1), (3, 3)`}

○ b. {`(0, 3), (1, 0), (2, 2), (3, 1)`}

○ c. {`(0, 2), (1, 1), (2, 0), (3, 3)`}

● d. {`(0, 1), (1, 2), (2, 3), (3, 0)`}

Q. 8 In the context of stable matchings, a valid partner for a man m is defined as:

● a. A woman w such that m and w are matched in at least one stable matching.

○ b. The woman w at the top of m's preference list.

○ c. Any woman w who accepts a proposal from m during the Gale-Shapley algorithm.

○ d. A woman w such that w and m are unmatched in all stable matchings.

Q. 9 Consider the following preferences of 5 men over 5 women, given as indices:

$$\begin{aligned} 0 &: [2, \mathbf{0}, 1, \mathbf{3}, 4] \\ 1 &: [1, 0, \mathbf{2}, 3, 4] \\ 2 &: [3, 0, \mathbf{1}, \mathbf{4}, 2] \\ 3 &: [\mathbf{3}, 1, \mathbf{2}, 0, 4] \\ 4 &: [\mathbf{4}, 3, \mathbf{2}, 1, 0] \end{aligned}$$

The preferences shown in **bold** indicate the valid partners for each man. Using the Gale-Shapley algorithm (where men propose), the resulting stable matches are:

○ a. $\{(0,2),(1,0),(2,1),(3,3),(4,4)\}$

○ b. $\{(0,3),(1,1),(2,0),(3,2),(4,4)\}$

● c. $\{(0,0),(1,2),(2,1),(3,3),(4,4)\}$

○ d. $\{(0,0),(1,2),(2,1),(3,0),(4,3)\}$

Q. 10 Consider the following preferences of 5 women over 5 men, given as indices:

$$\begin{aligned}
0 &: [\mathbf{2},\mathbf{0},1,\mathbf{3},4]\\
1 &: [1,\mathbf{0},2,3,\mathbf{4}]\\
2 &: [\mathbf{3},0,\mathbf{1},4,2]\\
3 &: [\mathbf{3},1,\mathbf{2},0,4]\\
4 &: [\mathbf{4},3,\mathbf{2},1,\mathbf{0}]
\end{aligned}$$

The preferences shown in **bold** indicate the valid partners for each woman. Using the Gale-Shapley algorithm (where men propose), the resulting stable matches are:

● a. $\{(0,3),(1,4),(2,1),(3,2),(4,0)\}$

○ b. $\{(0,2),(1,0),(2,1),(3,3),(4,4)\}$

○ c. $\{(0,3),(1,1),(2,0),(3,2),(4,4)\}$

○ d. $\{(0,0),(1,2),(2,1),(3,0),(4,3)\}$