Data Structures and Algorithms
with applications in Machine Learning
- MCQ 2 -

NAME: _____ **GROUP:** _____

Each Question: 1 Mark **Duration:** 20 Minutes

Completely fill the circles as shown: ○○●○

Answer sheet

Q1. ○ a. ○ b. ○ c. ○ d.

Q2. ○ a. ○ b. ○ c. ○ d.

Q3. ○ a. ○ b. ○ c. ○ d.

Q4. ○ a. ○ b. ○ c. ○ d.

Q5. ○ a. ○ b. ○ c. ○ d.

Q6. ○ a. ○ b. ○ c. ○ d.

Q7. ○ a. ○ b. ○ c. ○ d.

Q8. ○ a. ○ b. ○ c. ○ d.

Q9. ○ a. ○ b. ○ c. ○ d.

Q10. ○ a. ○ b. ○ c. ○ d.

The Quiz

Q. 1 According to the definition of an instability in a matching:

- ○ a. A pair (m, w) is unstable if the man m prefers another woman w' over w, but w' does not prefer m over her current match.
- ○ b. A matching is unstable if there are participants left unmatched.
- ○ c. A matching is unstable if every pair (m, w) prefers their current partners over all others.
- ○ d. A pair (m, w) is unstable if both m and w prefer each other over their current matches.

Q. 2 A matching $\phi : \mathcal{M} \to \mathcal{W}$ is considered stable if:

- ○ a. Every participant is matched to exactly one partner, regardless of preferences.
- ○ b. There is no instability, meaning no pair (m, w) exists such that m and w prefer each other over their current matches.
- ○ c. No man m prefers any woman w' over his current match, regardless of w''s preference.
- ○ d. The matching ensures that every participant is paired with their least preferred valid partner.

Q. 3 Regarding the *laissez-faire* approach to resolving instabilities:

- ○ a. It guarantees that the process will always terminate with a stable matching.
- ○ b. It resolves all instabilities simultaneously, ensuring optimal efficiency.
- ○ c. It resolves instabilities iteratively but does not guarantee convergence to a stable matching.
- ○ d. It avoids resolving instabilities and instead focuses on random pairings.

Q. 4 Which of the following statements about the Gale-Shapley algorithm is true?

- ○ a. The Gale-Shapley algorithm always results in the most optimal matching for both men and women simultaneously.
- ○ b. The Gale-Shapley algorithm terminates only if there are no instabilities but does not guarantee a stable matching.
- ○ c. The Gale-Shapley algorithm guarantees a stable matching for any set of preferences.
- ○ d. The Gale-Shapley algorithm ensures a matching, but stability depends on the initial preferences.

Q. 5 The following pseudo-code implements the Gale-Shapley algorithm:

Algorithm 1 Gale-Shapley Algorithm

Require: Lists of preferences for men $\mathcal{M}$ and women $\mathcal{W}$
Ensure: Stable matching ϕ

All men and women start as free
while $\exists$ free man m who has not proposed to every woman on his list **do**
    Pick such a man m
    Let w be the next woman on m's preference list
    if _____ **then** ▷ Fill in the blank
        Engage m and w
    else
        w rejects m
    end if
end while

What should replace the blank in the condition?

- ○ a. w is free or prefers m to her current match
- ○ b. w is free and prefers m to her current match
- ○ c. w prefers her current match to m
- ○ d. w is free but has already rejected m

Q. 6 The following function implements the Gale-Shapley stable matching algorithm.

```
def gale_shapley(men_prefs, women_prefs):
    """
    Implementation of Gale-Shapley stable matching algorithm.
    """
    n = len(men_prefs)
    free_men = list(range(n))
    proposals = np.zeros(n, dtype=int)
    women_partners = [-1] * n

    while free_men:
        m = free_men[0]
        w = men_prefs[m][proposals[m]]
        proposals[m] += 1

        if women_partners[w] == -1:
            women_partners[w] = m
            free_men.pop(0)
        else:
            current_m = women_partners[w]
            if __________:  # Fill in the blank
                women_partners[w] = m
                free_men.pop(0)
                free_men.append(current_m)

    matches = {(m, w) for w, m in enumerate(women_partners)}
    return matches
```

What should replace the blank?

- ○ a. `list(women_prefs[m]).index(w) < list(women_prefs[m]).index(current_m)`
- ○ b. `list(women_prefs[w]).index(m) < list(women_prefs[w]).index(current_m)`
- ○ c. `m > current_m`
- ○ d. `current_m == -1`

Q. 7 Consider the following preference lists for 4 men and 4 women:

```
men_prefs = [
    [0, 1, 2, 3],  # Man 0's preferences
    [2, 0, 1, 3],  # Man 1's preferences
    [1, 3, 0, 2],  # Man 2's preferences
    [0, 2, 3, 1]   # Man 3's preferences
]

women_prefs = [
    [2, 1, 3, 0],  # Woman 0's preferences
    [0, 3, 1, 2],  # Woman 1's preferences
    [1, 0, 3, 2],  # Woman 2's preferences
    [0, 1, 2, 3]   # Woman 3's preferences
]
```

What is the output of the Gale-Shapley algorithm (as a set of stable matches)?

- ○ a. {(0, 0), (1, 2), (2, 1), (3, 3)}
- ○ b. {(0, 3), (1, 0), (2, 2), (3, 1)}
- ○ c. {(0, 2), (1, 1), (2, 0), (3, 3)}
- ○ d. {(0, 1), (1, 2), (2, 3), (3, 0)}

Q. 8 In the context of stable matchings, a valid partner for a man m is defined as:

- ○ a. A woman w such that m and w are matched in at least one stable matching.
- ○ b. The woman w at the top of m's preference list.
- ○ c. Any woman w who accepts a proposal from m during the Gale-Shapley algorithm.
- ○ d. A woman w such that w and m are unmatched in all stable matchings.

Q. 9 Consider the following preferences of 5 men over 5 women, given as indices:

$$
\begin{aligned}
0 &: [2, \mathbf{0}, 1, \mathbf{3}, 4] \\
1 &: [1, 0, \mathbf{2}, 3, 4] \\
2 &: [3, 0, \mathbf{1}, \mathbf{4}, 2] \\
3 &: [\mathbf{3}, 1, \mathbf{2}, 0, 4] \\
4 &: [\mathbf{4}, 3, \mathbf{2}, 1, 0]
\end{aligned}
$$

The preferences shown in **bold** indicate the valid partners for each man. Using the Gale-Shapley algorithm (where men propose), the resulting stable matches are:

- ○ a. $\{(0,2),(1,0),(2,1),(3,3),(4,4)\}$
- ○ b. $\{(0,3),(1,1),(2,0),(3,2),(4,4)\}$
- ○ c. $\{(0,0),(1,2),(2,1),(3,3),(4,4)\}$
- ○ d. $\{(0,0),(1,2),(2,1),(3,0),(4,3)\}$

Q. 10 Consider the following preferences of 5 women over 5 men, given as indices:

$$
\begin{aligned}
0 &: [\mathbf{2}, \mathbf{0}, 1, \mathbf{3}, 4] \\
1 &: [1, \mathbf{0}, 2, 3, \mathbf{4}] \\
2 &: [\mathbf{3}, 0, \mathbf{1}, 4, 2] \\
3 &: [\mathbf{3}, 1, \mathbf{2}, 0, 4] \\
4 &: [\mathbf{4}, 3, \mathbf{2}, 1, \mathbf{0}]
\end{aligned}
$$

The preferences shown in **bold** indicate the valid partners for each woman. Using the Gale-Shapley algorithm (where men propose), the resulting stable matches are:

- ○ a. $\{(0,3),(1,4),(2,1),(3,2),(4,0)\}$
- ○ b. $\{(0,2),(1,0),(2,1),(3,3),(4,4)\}$
- ○ c. $\{(0,3),(1,1),(2,0),(3,2),(4,4)\}$
- ○ d. $\{(0,0),(1,2),(2,1),(3,0),(4,3)\}$